A variational multi-scale formulation for heterogeneous solids with nucleation and propagation of cohesive cracks

P.J. Sánchez[§‡], A.E. Huespe[§], S.M. Giusti[†], P.J. Blanco[†], R.A. Feijóo [†]

[§] *CIMEC, Centro Internacional de Métodos Computacionales en Ingeniería, Santa Fe, Argentina.*
[†] *LNCC, Laboratorio Nacional de Computación Científica, Petrópolis, Brasil.*
[‡] *GIMNI, Universidad Tecnológica Nacional, FRSF, Santa Fe, Argentina.*

Abstract: This contribution presents a variational multi-scale formulation to describe the constitutive response of heterogeneous solids containing multiple cohesive cracks. It considers two coupled mechanical problems at different physical length scales, typically denoted as *macro* and *micro* scales where, in both of them, all the formalisms and concepts of continuum mechanics are applicable. The methodology is based on the classical volume averaging technique of the microscopic strain and stress tensor fields over a local *Representative Volume Element (RVE)*. The proposed variational formulation follows the general axiomatic framework reported in a previous contribution, see E.A. de Souza Neto & R.A. Feijóo 2008. The mechanical response of each phase of the *micro* heterogeneous medium (typically a matrix phase with particulate aggregates) is described by means of local continuum dissipative theories which, as well, can induce material instabilities and strain localization patterns. During the load history, localized failure in the form of multiple cracks can be nucleated and propagated in the (initially continuum) matrix phase. The generation and accumulation of this kind of *micro*-defects have important implicances at the *macro* constitutive level and, therefore, they are included in the *RVE* model. In this context of analysis, the presence of cracks introduces two key ingredients in the definition of the present formulation: *(i)* an enhanced kinematical description which is capable to reproduce jumps or discontinuities in the displacement field (commonly known as *Strong Discontinuities Kinematics*) via the introduction of a new kinematical field, and *(ii)* cohesive-type constitutive laws governing the evolution of such enriched discontinuous modes. The theoretical consequences about the incorporation of this particular kinematics are analyzed in detail throughout the paper, being this topic the main objective of the work. Following a consistent variational approach, the spaces of minimal kinematical restrictions over the *RVE* model are clearly defined, in order to preserve the homogenization concept of strain tensor. Different models can be deduced and formulated from the obtained minimally constrained kinematics, in fact, the classical multi-scale models are recovered by adequate specialization of such spaces. The energy consistency between *micro* and *macro* scales is guaranteed by the introduction of the Hill-Mandel principle of Macro Homogeneity in the governing variational principle of the equilibrium at the *RVE*. The proposed methodology is sufficiently general to consider multiple strong discontinuities at the *micro* scale as well as for inducing a weak discontinuity mode at the *macro* scale, similar to the methodology proposed by Belytschko et. al 2008.

Keywords: variational multi-scale models, material failure mechanics, cohesive cracks, strong discontinuity kinematics.